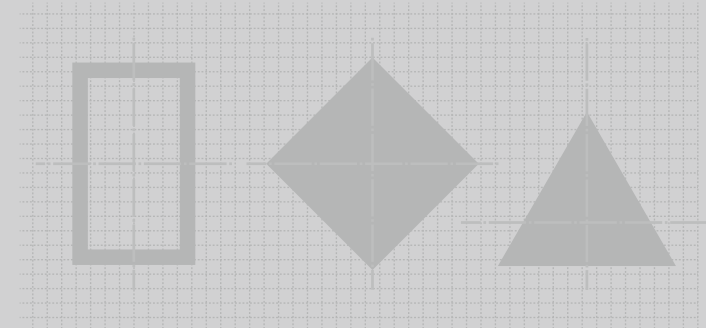


3. 力学



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3-1. 平面図形

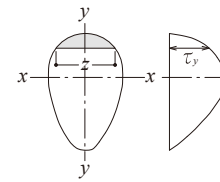
断面の諸係数の定義

断面積	$A = \int_A dA$	
断面1次モーメント	$S_x = \int_A y dA$, $S_y = \int_A x dA$	
図心	$\bar{y} = \frac{S_x}{A}$, $\bar{x} = \frac{S_y}{A}$	
断面2次モーメント	$I_x = \int_A y^2 dA$, $I_y = \int_A x^2 dA$	
断面相乗モーメント	$I_{xy} = \int_A xy dA$	
断面極2次モーメント	$I_p = \int_A r^2 dA$, $I_p = I_x + I_y$	
断面2次半径	$i_x = \sqrt{\frac{I_x}{A}}$, $i_y = \sqrt{\frac{I_y}{A}}$	
断面極2次半径	$i_p = \sqrt{\frac{I_p}{A}}$	
断面係数	$Z_1 = \frac{I}{y_1}$, $Z_2 = \frac{I}{y_2}$	
軸の平行移動	$S'_x = S_x + Ay_0$, $S'_y = S_y + Ax_0$ $I'_x = I_x + 2y_0 S_x + Ay_0^2$, $I'_y = I_y + 2x_0 S_y + Ax_0^2$ $I'_{xy} = I_{xy} + x_0 S_x + y_0 S_y + Ax_0 y_0$ 0を図心にすれば $I'_x = I_x + Ay_0^2$, $I'_y = I_y + Ax_0^2$ $I'_{xy} = I_{xy} + Ax_0 y_0$	
軸の回転移動	$S'_x = S_x \cos\alpha - S_y \sin\alpha$ $S'_y = S_x \sin\alpha + S_y \cos\alpha$ $I'_x = I_x \cos^2\alpha + I_y \sin^2\alpha - I_{xy} \sin 2\alpha$ $I'_y = I_x \sin^2\alpha + I_y \cos^2\alpha + I_{xy} \sin 2\alpha$ $I'_{xy} = \frac{I_x - I_y}{2} \sin 2\alpha + I_{xy} \cos 2\alpha$	
主軸および主断面2次モーメント	$I'_{xy} = 0$ となるとき、 x' , y' を主軸という 主軸の傾き $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x}$ $I_x = \frac{1}{2} (I_x + I_y) + \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$ $I_y = \frac{1}{2} (I_x + I_y) - \frac{1}{2} \sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$ $I'_x = I_x \cos^2\alpha + I_y \sin^2\alpha$ $I'_y = I_x \sin^2\alpha + I_y \cos^2\alpha$ $I'_{xy} = \frac{I_x - I_y}{2} \sin 2\alpha$	

断面性能算出公式

断面	面積 A	図心より縁に至る距離 y_0	断面2次モーメント I_x	断面2次半径 i_x	断面係数 Z_x
	bh	$\frac{h}{2}$	$\frac{bh^3}{12}$	$\frac{h}{\sqrt{12}} = 0.289h$	$\frac{bh^2}{6}$
	bh	$y_0 = \frac{bh}{\sqrt{b^2+h^2}}$ $x_0 = \frac{1}{2} \sqrt{b^2+h^2}$	$\frac{b^3 h^3}{6(b^2+h^2)}$	$\frac{bh}{\sqrt{6(b^2+h^2)}}$	$\frac{b^2 h^2}{6\sqrt{b^2+h^2}}$
	$\frac{\pi}{4} d^2 = 0.785d^2$	$\frac{d}{2}$	$\frac{\pi}{64} d^4 = 0.0491d^4$	$\frac{d}{4}$	$\frac{\pi}{32} d^3 = 0.0982d^3$
	$\frac{\pi}{4} (d^2 - d_1^2) = 0.785 \cdot (d^2 - d_1^2)$	$\frac{d}{2}$	$\frac{\pi}{64} (d^4 - d_1^4) = 0.0491 \cdot (d^4 - d_1^4)$	$\frac{\sqrt{d^2 + d_1^2}}{4}$	$\frac{\pi}{32} \frac{d^4 - d_1^4}{d} = 0.0982 \cdot \frac{d^4 - d_1^4}{d}$
	πab	a	$\frac{\pi ba^3}{4}$	$\frac{a}{2}$	$\frac{\pi ba^2}{4}$
	$\frac{\pi (BH - bh)}{4}$	$\frac{H}{2}$	$\frac{\pi (BH^3 - bh^3)}{64}$	$\sqrt{\frac{BH^3 - bh^3}{16(BH - bh)}}$	$\frac{\pi}{32H} (BH^3 - bh^3)$
	$\frac{\pi}{4} r^2$	$y_1 = 0.4244r$ $y_2 = 0.5756r$	$0.055r^4$	$0.2643r$	$Z_1 = 0.1296r^3$ $Z_2 = 0.0956r^3$
	$0.2146r^2$	$y_1 = 0.2234r$ $y_2 = 0.7766r$	$0.0075r^4$	$0.187r$	$Z_1 = 0.03357r^3$ $Z_2 = 0.00966r^3$

せん断応力



一般に $\tau_y = \frac{QS_y}{bI_x}$

$\tau_{max} = \kappa \frac{Q}{A}$: 最大せん断応力度

ここに
 S_y : ハッチ部分の断面1次モーメント
 I_x : 断面2次モーメント

$\kappa = \frac{\text{最大応力度}}{\text{平均応力度}}$

$y_i =$ 中立軸からの距離

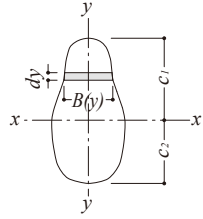
断面	面積 A	図心より縁に至る距離 y_0	断面2次モーメント I_x	断面2次半径 i_x	断面係数 Z_x
	$bh - b_i h_i$	$\frac{h}{2}$	$\frac{bh^3 - b_i h_i^3}{12}$	$\sqrt{\frac{bh^3 - b_i h_i^3}{12(bh - b_i h_i)}}$	$\frac{bh^3 - b_i h_i^3}{6h}$
	$bh - w(b-t)$	$\frac{b}{2}$	$\frac{2fb^3 + wt^3}{12}$	$\sqrt{\frac{2fb^3 + wt^3}{12\{bh - w(b-t)\}}}$	$\frac{2fb^3 + wt^3}{6b}$
	$bf + wt$	$y_1 = \frac{th^2 + f^2(b-t)}{2(bf + wt)}$ $y_2 = h - y_1$	$\frac{th^3 + (b-t)f^3}{3} - Ay_1^2$	$\sqrt{\frac{I_x}{A}}$	$Z_1 = \frac{I_x}{y_1}$ $Z_2 = \frac{I_x}{y_2}$
	$b(d - d_1)$	$\frac{d}{2}$	$\frac{b(d^3 - d_1^3)}{12}$	$\sqrt{\frac{d^3 - d_1^3}{12(d - d_1)}}$ $= 0.289\sqrt{d^2 - dd_1 + d_1^2}$	$\frac{b(d^3 - d_1^3)}{6d}$

断面の核

<p>矩形</p> <p>$e = d/3$ $r = \frac{bd}{3\sqrt{b^2 + d^2}}$</p>	<p>中空正方形</p> <p>$e = \frac{11}{3} \left\{ 1 + \left(\frac{d_2}{d_1} \right)^2 \right\}$ $r = 0.2358d_1 \left\{ 1 + \left(\frac{d_2}{d_1} \right)^2 \right\}$</p>	<p>三角形</p> <p>$e = d/4$ $r_1 = d/6$ $r_2 = d/12$</p>
<p>中空円</p> <p>$e = \frac{D_1}{4} \left\{ 1 + \left(\frac{D_2}{D_1} \right)^2 \right\}$</p>	<p>円</p> <p>$e = D/4$</p>	<p>I形</p> <p>$e = \frac{2i^2}{h}$ $e = \frac{2i^2}{b}$</p>

断面形	せん断力 τ	κ
<p>1. 矩形</p>	$\tau = \frac{3}{2} \frac{Q}{bh} \left\{ 1 - \left(\frac{2y_i}{h} \right)^2 \right\}$ $\tau_{max} = \frac{3}{2} \frac{Q}{bh} = \frac{3}{2} \frac{Q}{A}$ $(y_i = 0)$	$\frac{3}{2}$
<p>2. 正方形</p>	$\tau = \frac{Q}{a^2} \left\{ 1 + \sqrt{2} \frac{y_i}{a} - 4 \left(\frac{y_i}{a} \right)^2 \right\}$ $\tau_{max} = \frac{9}{8} \frac{Q}{a^2} = 1.125 \frac{Q}{A}$ $(y_i = \frac{1}{4} e = \frac{\sqrt{2}}{8} a)$	$\frac{9}{8}$
<p>3. 円</p>	$\tau = \frac{4}{3} \frac{Q}{\pi r^2} \left\{ 1 - \left(\frac{y_i}{r} \right)^2 \right\}$ $\tau_{max} = \frac{4}{3} \frac{Q}{\pi r^2} = \frac{4}{3} \frac{Q}{A}$ $(y_i = 0)$	$\frac{4}{3}$
<p>4. 薄肉パイプ</p>	$\tau = \frac{Q}{\pi r t} \left\{ 1 - \left(\frac{y_i}{r} \right)^2 \right\}$ $\tau_{max} = \frac{Q}{\pi r t} = 2 \frac{Q}{A}$ $(y_i = 0)$	2
<p>6. パイプ</p>	$r_2 \geq y_i \geq r_1$ $\tau = \frac{4}{3} \frac{Q}{\pi(r_2^4 - r_1^4)} (r_2^2 - y_i^2)$ $r_1 \geq y_i$ $\tau = \frac{4}{3} \frac{Q}{\pi(r_2^4 - r_1^4)} \left\{ r_2^2 + r_1^2 - 2y_i^2 + \sqrt{(r_2^2 - y_i^2)(r_1^2 - y_i^2)} \right\}$	$\kappa = \frac{4(r_2^2 + r_2 r_1 + r_1^2)}{3(r_2^2 + r_1^2)}$ $\tau_{max} = \frac{4}{3} \frac{Q(r_2^2 + r_2 r_1 + r_1^2)}{\pi(r_2^4 - r_1^4)}$ $= \frac{Q}{A} \frac{4(r_2^2 + r_2 r_1 + r_1^2)}{3(r_2^2 + r_1^2)}$
<p>8. 対称形断面</p>	$\frac{h_2}{2} \geq y_i \geq \frac{h_1}{2}$ $\tau = \frac{3Q}{2(b_2 h_2^3 - b_1 h_1^3)} (h_2^2 - 4y_i^2)$ $\frac{h_1}{2} \geq y_i$ $\tau = \frac{3Q}{2(b_2 h_2^3 - b_1 h_1^3)} \left(\frac{b_2 h_2^2 - b_1 h_1^2}{b_2 - b_1} - 4y_i^2 \right)$	$\kappa = \frac{3(b_2 h_2^2 - b_1 h_1^2)(b_2 h_2 - b_1 h_1)}{2(b_2 h_2^3 - b_1 h_1^3)(b_2 - b_1)}$ $\tau_{max} = \frac{b_2 h_2^2 - b_1 h_1^2}{(b_2 h_2^3 - b_1 h_1^3)(b_2 - b_1)} \frac{3Q}{2}$

塑性断面係数



$$Z_p = \int_0^{c_1} B(y)y dy + \int_0^{c_2} B(y)y dy$$

$B(y)$: 中立軸から距離 y だけ離れたところの断面幅
 c_1, c_2 : それぞれ中立軸から上下縁までの距離

断面形	塑性断面係数 Z_p	断面形	塑性断面係数 Z_p
 矩形	$B \cdot H^2 / 4$	 楕円	$B \cdot H^2 / 6$
 菱形	$B \cdot H^2 / 12$	 斜正方形	$\sqrt{2} a^3 / 6$
 円	$d^3 / 6, \frac{4}{3} R^3$	 正三角形	$\frac{2 - \sqrt{2}}{6} B \cdot H^2$
 中空円	$\frac{4}{3} R^3 \left(1 - \left(\frac{T}{R}\right)^3\right)$	 薄肉中空円	$4 \cdot R_m^2 \cdot T$
 中空矩形	$B \cdot T_2(H - T_2) + \frac{1}{2}(H - 2T_2)^2 T_1$	 薄肉中空矩形	$A_f \cdot H_f + \frac{1}{4} A_w \cdot H_f$ A_f : 片側フランジプレートの断面積 A_w : ウェブプレートの全断面積
 厚肉I形	$B \cdot T_f(H - T_f) + \frac{1}{4}(H - 2T_f)^2 T_w + 0.4292R^2(H - 2T_f - 0.4467R)$	 薄肉I形	$A_f \cdot H_f + \frac{1}{4} A_w \cdot H_f$ A_f : 片側フランジの断面積 A_w : ウェブの全断面積
 厚肉I形弱軸	$\frac{1}{2} B^2 \cdot T_f + \frac{1}{4} (H - 2T_f) \cdot T_w^2 + 0.4292R^2(T_w + 0.4467R)$	 薄肉I形弱軸	$\frac{1}{2} A_f \cdot B + \frac{1}{4} A_w \cdot T_w$ A_f : 片側フランジの断面積 A_w : ウェブの全断面積

純ねじり・曲げねじりに関する断面の定数

【純ねじり：反り拘束を無視した場合】 【曲げねじり：反り拘束を考慮した場合】

$M_T = GJ \frac{d\phi}{dx}$ $\tau_{max} = M_T \frac{t}{J}$ $M_T = GJ \frac{d\phi}{dx} - EI_w \frac{d^3\phi}{dx^3}$

ϕ : ねじり角 G : せん断弾性係数
 O' : 断面中心 τ_{max} : 最大ねじりせん断応力 EI_w : 曲げねじり剛性
 t : 板厚 GJ : ねじり剛性 I_w : 曲げねじり定数
 M_T : ねじりモーメント J : サンプソンのねじり定数 E : ヤング係数

断面形	x_0	y_0	I_w	J
 S : せん断中心 O : 図心	0	$-e = -\frac{\sqrt{2}}{4} b$	$\frac{(tb)^3}{18}$	$\frac{2}{3} bt^3$
	e_1	$-e_2$	$\frac{t^3}{36} (b_1^3 + b_2^3)$	$\frac{t^3}{3} (b_1 + b_2)$
	0	$-e = -\frac{t_3 h^2}{2A}$	$\frac{(t_1 b)^3}{144} + \frac{(t_3 h)^3}{36}$	$\frac{1}{3} (bt_1^3 + ht_3^3)$
	0	$-\frac{e_1 I_1 - e_2 I_2}{I_1 + I_2}$ $= -\left(e_1 - \frac{t_2 b_2^3}{t_1 b_1^3 + t_2 b_2^3} h\right)$	$\frac{I_1 I_2}{I_1 + I_2} h^2$ $= \frac{h^2}{12} \cdot \frac{t_1 b_1^3 t_2 b_2^3}{t_1 b_1^3 + t_2 b_2^3}$	$\frac{b_1 t_1^3 + b_2 t_2^3 + ht_3^3}{3}$
	0	$-\left(e + \frac{I_1}{T_y} h\right)$ $= -\left(e + \frac{3t_1 h^2}{6t_1 h + t_3 b}\right)$	$\frac{I_1^2 + 2I_1 I_3}{I_1} \cdot \frac{h^2}{3}$ $= \frac{t_1 h^3 b^2}{12} \cdot \frac{3t_1 h + 2}{6t_1 h + t_3 b}$	$\frac{2ht_1^3 + bt_3^3}{3}$

3-2. はりおよびラーメンの応力計算公式

はりの応力計算公式

荷重形式	単純支持 \triangle ----- \triangle			両端固定 ∇ ----- ∇		
	反力 R	曲げモーメント M_0	最大たわみ δ_{max}	反力 R	固定端モーメント C_A, C_B	最大たわみ δ_{max}
	$R_A = R_B = \frac{P}{2}$	$M_0 = \frac{P\ell}{4}$	$\delta_{max} = \frac{P\ell^3}{48EI}$	$R_A = R_B = \frac{P}{2}$	$C_A = -C_B = -\frac{P\ell}{8}$	$\delta_{max} = \frac{P\ell^3}{192EI}$
	$R_A = \frac{Pb}{\ell}$ $R_B = \frac{Pa}{\ell}$	$M_C = \frac{Pab}{\ell}$	$\delta_{max} = \frac{Pb(\ell^2 - b^2)^{3/2}}{9\sqrt{3}EI\ell}$ ($a > b$ のとき) $x = \sqrt{\frac{\ell^2 - b^2}{3}}$ $\delta_c = \frac{Pa^2b^2}{3EI\ell}$	$R_A = \frac{Pb^2}{\ell^2}(3a+b)$ $R_B = \frac{Pa^2}{\ell^2}(3b+a)$	$C_A = -\frac{Pab^2}{\ell^2}$ $C_B = \frac{Pa^2b}{\ell^2}$	$\delta_{max} = \frac{2Pa^3b^2}{3EI(3a+b)^2}$ ($x = \frac{2a\ell}{3a+b}$)
	$R_A = R_B = P$	$M_0 = \frac{P\ell}{3}$	$\delta_{max} = \frac{23}{648} \frac{P\ell^3}{EI}$	$R_A = R_B = P$	$C_A = -C_B = -\frac{2P\ell}{9}$	$\delta_{max} = \frac{5}{648} \frac{P\ell^3}{EI}$
	$R_A = R_B = \frac{3P}{2}$	$M_0 = \frac{P\ell}{2}$	$\delta_{max} = \frac{19}{384} \frac{P\ell^3}{EI}$	$R_A = R_B = \frac{3P}{2}$	$C_A = -C_B = -\frac{5P\ell}{16}$	$\delta_{max} = \frac{1}{96} \frac{P\ell^3}{EI}$
	$R_A = R_B = \frac{w\ell}{2}$	$M_0 = \frac{w\ell^2}{8}$	$\delta_{max} = \frac{5}{384} \frac{w\ell^4}{EI}$	$R_A = R_B = \frac{w\ell}{2}$	$C_A = -C_B = -\frac{w\ell^2}{12}$	$\delta_{max} = \frac{1}{384} \frac{w\ell^4}{EI}$
	$R_A = wb \frac{2c+b}{2\ell}$ $R_B = wb \frac{2a+b}{2\ell}$	$M_{max} = R_A(a + \frac{R_A}{2w})$ ($x = a + R_A/w$)	$\delta_c = \frac{wb}{48EI\ell} \left[\{(\ell+a-c)(\ell-a+c) - \frac{b^2}{4}\}^2 + \frac{b^3(2\ell-b)}{16} \right]$	$R_A = \frac{wb}{2\ell^2} \{ (b+2c)\ell^2 - (a-c)(2ac+bc+ab) \}$ $R_B = wb - R_A$	$C_A = -\frac{wb}{8\ell^2} \{ (b+2c)^2(2a+b) + \frac{1}{3}b^2(2\ell-6c-3b) \}$ $C_B = \frac{wb}{8\ell^2} \{ (2a+b)^2(b+2c) + \frac{1}{3}b^2(2\ell-6a-3b) \}$	$\delta_x = \frac{1}{6EI} \{ 3C_Ax^2 - R_Ax^3 + \frac{w}{4}(x-a)^4 \}$
	$R_A = \frac{w\ell}{6}$ $R_B = \frac{w\ell}{3}$	$M_{max} = 0.064 w\ell^2$ ($x = 0.577\ell$)	$\delta_{max} = 0.00652 \frac{w\ell^4}{EI}$ ($x = 0.519\ell$)	$R_A = \frac{3w\ell}{20}$ $R_B = \frac{7w\ell}{20}$	$C_A = -\frac{w\ell^2}{30}, C_B = \frac{w\ell^2}{20}$ $M_{max} = 0.0215w\ell^2$ ($x = 0.548\ell$)	$\delta_{max} = 0.00131 \frac{w\ell^4}{EI}$ ($x = 0.525\ell$)
	$R_A = R_B = \frac{w\ell}{4}$	$M_0 = \frac{w\ell^2}{12}$	$\delta_{max} = \frac{1}{120} \frac{w\ell^4}{EI}$	$R_A = R_B = \frac{w\ell}{4}$	$C_A = -C_B = -\frac{5w\ell^2}{96}$ $M_{max} = \frac{w\ell^2}{32}$	$\delta_{max} = \frac{7}{3840} \frac{w\ell^4}{EI}$
	$R_A = R_B = \frac{w(\ell-a)}{2}$	$M_0 = \frac{w}{24} (3\ell^2 - 4a^2)$	$\delta_{max} = \frac{w}{1920EI} (5\ell^2 - 4a^2)^2$	$R_A = R_B = \frac{w(\ell-a)}{2}$	$C_A = -C_B = -\frac{w}{12} (\ell^2 - 2a^2 + \frac{a^3}{\ell})$	$\delta_{max} = \frac{w}{1920EI} (5\ell^4 - 20\ell a^3 + 16a^4)$
	$R_A = R_B = \frac{w\ell}{4}$	$M_C = \frac{w\ell^2}{16}$ $M_{D,E} = \frac{5w\ell^2}{96}$	$\delta_{max} = \frac{7}{1024} \frac{w\ell^4}{EI}$	$R_A = R_B = \frac{w\ell}{4}$	$C_A = -C_B = -\frac{17}{384} w\ell^2$ $M_C = \frac{7}{384} w\ell^2$	$\delta_{max} = \frac{w\ell^4}{768EI}$
	$R_A = R_B = \frac{w\ell}{4}$	$M_C = \frac{7w\ell^2}{108}$	$\delta_{max} = \frac{259}{38880} \frac{w\ell^4}{EI}$	$R_A = R_B = \frac{w\ell}{4}$	$C_A = -C_B = -\frac{37}{864} w\ell^2$ $M_C = \frac{19}{864} w\ell^2$	$\delta_{max} = \frac{407}{311040} \frac{w\ell^4}{EI}$
	$R_A = -R_B = \frac{M}{\ell}$	$a > b$ のとき $M_C = \frac{Ma}{\ell}$	$a > b$ のとき $\delta_{max} = \frac{M(\ell^2 - 3b^2)^{3/2}}{9\sqrt{3}EI\ell}$ ($x = \sqrt{\frac{\ell^2 - 3b^2}{3}}$)	$R_A = -R_B = \frac{6abM}{\ell^3}$	$C_A = -\frac{bM}{\ell^2}(2\ell - 3b)$ $C_B = -\frac{aM}{\ell^2}(2\ell - 3a)$	$\ell < 3a$ のとき $\delta_{max} = \frac{b(2a-b)^3M}{54a^2EI}$ $x = \ell(2a-b)/3a$
	$R_A = -R_B = \frac{M}{\ell}$	$M_x = -M(1 - \frac{x}{\ell})$	$\delta_{max} = \frac{M\ell^2}{9\sqrt{3}EI}$ ($x = (1 - \frac{1}{\sqrt{3}})\ell$)	-	-	-

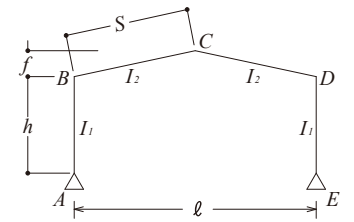
はりの応力計算公式

荷重形式	反力 R	曲げモーメント M	最大たわみ δ_{max}
	$R_A=R_C=\frac{3}{8}w\ell$ $R_B=\frac{5}{4}w\ell$	$M_B=-\frac{1}{8}w\ell^2$ $M_D=\frac{9}{128}w\ell^2$ ($x=\frac{3}{8}\ell$)	$\delta_{max}=\frac{w\ell^4}{185EI}$ ($x=0.422\ell$)
	$R_A=\frac{5}{16}P$ $R_C=\frac{5}{16}P$ $R_B=\frac{11}{8}P$	$M_B=-\frac{3}{16}P\ell$ $M_D=M_E=\frac{5}{32}P\ell$	$\delta_{max}=\frac{P\ell^3}{48\sqrt{5}EI}$ ($x=\frac{1}{\sqrt{5}}\ell$) $\delta_D=\delta_E=\frac{7P\ell^3}{768EI}$
	$R_A=R_C=\frac{2}{3}P$ $R_B=\frac{8}{3}P$	$M_B=-\frac{1}{3}P\ell$ $M_E=M_F=\frac{1}{9}P\ell$ $M_D=M_G=\frac{2}{9}P\ell$	$\delta_D=\delta_G=\frac{7P\ell^3}{486EI}$
	$R_A=\frac{7}{16}w\ell$ $R_C=-\frac{1}{16}w\ell$ $R_B=\frac{5}{8}w\ell$	$M_B=-\frac{1}{16}w\ell^2$ $M_D=\frac{49}{512}w\ell^2$ ($x=\frac{7}{16}\ell$)	$\delta_D=\frac{7w\ell^4}{768EI}$ ($x=\frac{1}{2}\ell$)
	$R_A=\frac{1}{16}(7w_1-w_2)\ell$ $R_C=\frac{1}{16}(7w_2-w_1)\ell$ $R_B=\frac{5}{8}(w_1+w_2)\ell$	$M_B=-\frac{1}{16}(w_1+w_2)\ell^2$	$\delta_D=\frac{1}{768EI}(7w_1-3w_2)\ell^4$ $\delta_E=\frac{1}{768EI}(7w_2-3w_1)\ell^4$
	$W=w_1l_1+w_2l_2$ $R_B=W-R_A-R_C$ $R_A=\frac{w_1l_1}{2}-\frac{1}{8(1+\alpha)l_1}$ ($\alpha w_1l_1^2+w_2l_2^2$) $R_C=\frac{w_2l_2}{2}-\frac{1}{8(1+\alpha)l_2}$ ($\alpha w_1l_1^2+w_2l_2^2$)	$M_B=-\frac{1}{8(1+\alpha)}$ ($\alpha w_1l_1^2+w_2l_2^2$)	$\delta_D=\frac{5w_2l_2^4}{384EI_2}-\frac{M_Bl_2^2}{16EI_2}$ (BC ばり中央部)

片持ちはりの応力計算公式

荷重形式	反力 R	曲げモーメント M	最大たわみ δ_{max}
	$R_B=P$	$M_B=-P\ell$	$\delta_A=\frac{1}{3}\frac{P\ell^3}{EI}$
	$R_B=P$	$M_B=-Pb$	$\delta_A=\frac{1}{6}\frac{P}{EI}(3b^2\ell-b^3)$
	$R_B=w\ell$	$M_B=-\frac{1}{2}w\ell^2$	$\delta_A=\frac{1}{8}\frac{w\ell^4}{EI}$
	$R_B=\frac{w\ell}{2}$	$M_B=-\frac{1}{6}w\ell^2$	$\delta_A=\frac{1}{30}\frac{w\ell^4}{EI}$
	$R_B=\frac{w\ell}{2}$	$M_B=-\frac{1}{3}w\ell^2$	$\delta_A=\frac{11}{120}\frac{w\ell^4}{EI}$
	$R_B=0$	$M_B=M$	$\delta_A=-\frac{1}{2}\frac{M\ell^2}{EI}$
	$R_A=\frac{3}{8}w\ell$ $R_B=\frac{5}{8}w\ell$	$M_{max}=\frac{9}{128}w\ell^2$ ($x=\frac{3}{8}\ell$) $M_B=-\frac{1}{8}w\ell^2$	$\delta_{max}=0.00541\frac{w\ell^4}{EI}$ ($x=0.4215\ell$)
	$R_A=\frac{1}{10}w\ell$ $R_B=\frac{2}{5}w\ell$	$M_{max}=0.0298w\ell^2$ ($x=0.4474\ell$) $M_B=-\frac{1}{15}w\ell^2$	$\delta_{max}=0.002385\frac{w\ell^4}{EI}$ ($x=0.4472\ell$)
	$R_A=\frac{11}{40}w\ell$ $R_B=\frac{9}{40}w\ell$	$M_{max}=0.0423w\ell^2$ ($x=0.329\ell$) $M_B=-\frac{7}{120}w\ell^2$	$\delta_{max}=0.003045\frac{w\ell^4}{EI}$ ($x=0.402\ell$)
	$R_A=-R_B=\frac{3}{2}\frac{M}{\ell}$	$M_{max}=M$ $M_B=-\frac{1}{2}M$	$\delta_{max}=\frac{1}{27}\frac{M\ell^2}{EI}$ ($x=\frac{1}{3}\ell$)
	$R_A=\frac{5}{16}P$ $R_B=\frac{11}{16}P$	$M_C=\frac{5}{32}P\ell$ $M_B=-\frac{3}{16}P\ell$	$\delta_{max}=0.00932\frac{P\ell^3}{EI}$ ($x=0.4472\ell$)

柱脚ピン対称山形ラーメン



$$K_1 = \frac{I_1}{h}$$

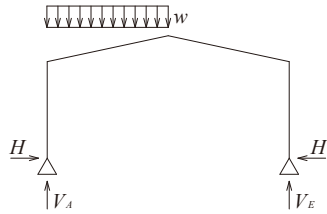
$$K_2 = \frac{I_2}{S}$$

$$k = \frac{K_2}{K_1}$$

$$a = h^2(k+3) + f(3h+f)$$

荷重状態

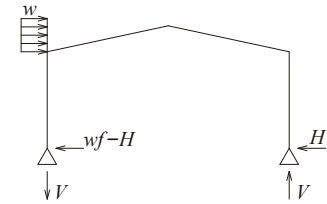
反力



$$V_A = \frac{3}{8}w\ell$$

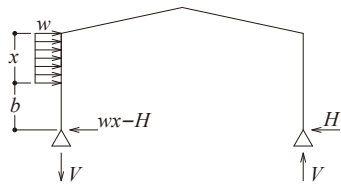
$$V_E = \frac{1}{8}w\ell$$

$$H = \frac{w\ell^2}{64} \frac{8h+5f}{a}$$



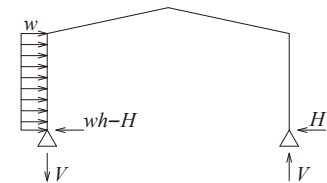
$$V = \frac{wf}{\ell} \left(h + \frac{f}{2} \right)$$

$$H = \frac{wf}{16} \frac{8h^2(k+3) + 5f(4h+f)}{a}$$



$$V = wx \frac{(h - \frac{x}{2})}{\ell}$$

$$H = \frac{wx(h+b)}{16h} \frac{k(5h^2 - b^2) + 6h(2h+f)}{a}$$

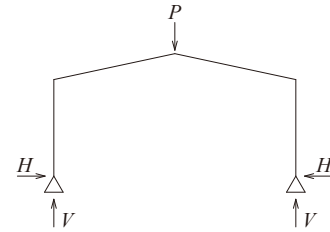


$$V = \frac{wh^2}{2\ell}$$

$$H = \frac{wh^2}{16} \frac{5kh + 6(2h+f)}{a}$$

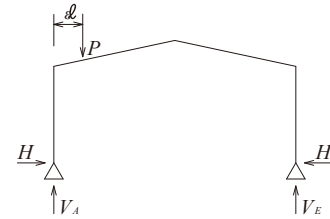
荷重状態

反力



$$V = \frac{P}{2}$$

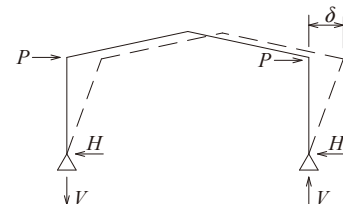
$$H = \frac{P\ell}{8} \frac{3h+2f}{a}$$



$$V_A = (1-\varepsilon)P$$

$$V_E = \varepsilon P$$

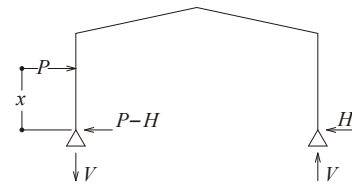
$$H = \frac{P\ell}{4} \cdot \varepsilon \frac{6h(1-\varepsilon) + f(3-4\varepsilon)}{a}$$



$$V = \frac{2Ph}{\ell}$$

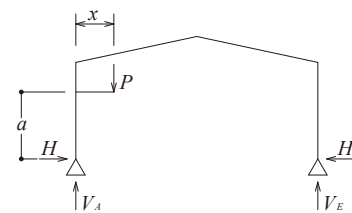
$$\delta = \frac{Ph^2}{12EK_I} \frac{4(k+1)}{k}$$

$$H = P$$



$$V = \frac{Px}{\ell}$$

$$H = \frac{Px}{4} \frac{k(3h - \frac{x^2}{h}) + 3(2h+f)}{a}$$



$$V_A = P \frac{\ell-x}{\ell}$$

$$V_E = \frac{Px}{\ell}$$

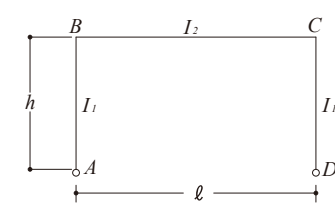
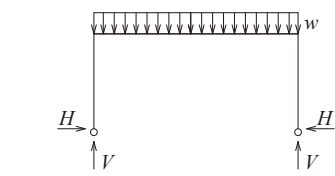
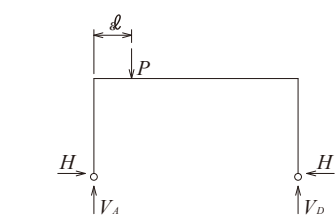
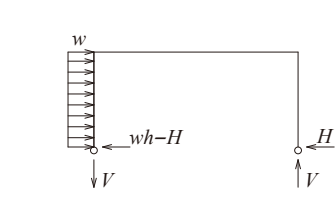
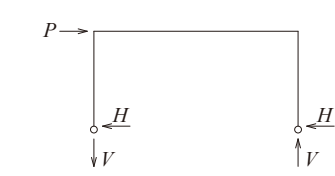
$$H = \frac{3Px}{4h} \frac{k(h^2 - a^2) + h(2h+f)}{a}$$

柱脚固定対称山形ラーメン

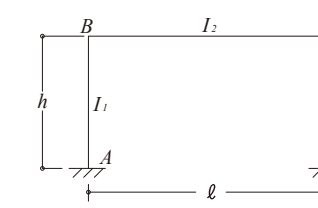
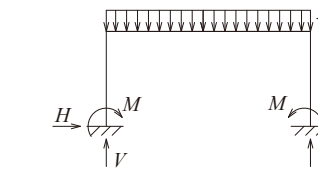
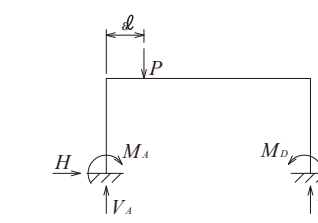
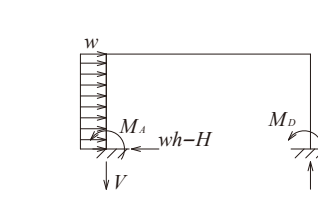
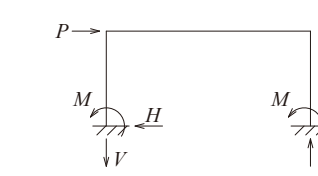
	$K_1 = \frac{I_1}{h}$ $K_2 = \frac{I_2}{S}$ $k = \frac{K_2}{K_1}$ $\beta = (kh+f)^2 + 4k(h^2 + hf + f^2)$
荷重状態	反力
	$V_E = \frac{w\ell}{32} \frac{3(4k+1)}{3k+1} \quad V_A = \frac{w\ell}{2} - V_E$ $H = \frac{w\ell^2}{16} \frac{k(4h+5f)+f}{\beta}$ $M_A = \frac{w\ell^2}{96} \left\{ \frac{kh(8h+15f)+f(6h-f)}{\beta} - \frac{3}{2(3k+1)} \right\}$ $M_E = \frac{w\ell^2}{96} \left\{ \frac{kh(8h+15f)+f(6h-f)}{\beta} + \frac{3}{2(3k+1)} \right\}$
	$V = \frac{wf}{8} \frac{12k(h+f)+5f}{\ell(3k+1)}$ $H = \frac{wf}{4} \frac{2kh^2(k+4)+10khf+f^2(5k+1)}{\beta}$ $M_A = \frac{wf}{24} \left[\frac{12h(3k+2)+3f}{6k+2} + \frac{f\{kh(4h+9f)+f(6h+f)\}}{\beta} \right]$ $M_E = \frac{wf}{24} \left[\frac{12h(3k+2)+3f}{6k+2} - \frac{f\{kh(4h+9f)+f(6h+f)\}}{\beta} \right]$
	$V = \frac{wa^3}{2\ell h} \frac{k}{3k+1}$ $H = \frac{wa^3 k}{4h} \frac{2h(k+2)+2f-a(k+1)}{\beta}$ $M_A = \frac{wa^2}{24h} \left\{ -\frac{2h^2 k(12h+3hk-4ak-12a+18f)}{\beta} - \frac{3a^2 k(kh+2h+f)+8fk(3hf-af-3ha)+6hf^2}{\beta} - \frac{6h+6k(3h-a)}{3k+1} \right\}$ $M_E = \frac{wa^2}{24h} \left\{ -\frac{2h^2 k(12h+3hk-4ak-12a+18f)}{\beta} - \frac{3a^2 k(kh+2h+f)+8fk(3hf-af-3ha)+6hf^2}{\beta} + \frac{6h+6k(3h-a)}{3k+1} \right\}$
	$V = \frac{wh^2}{2\ell} \frac{k}{3k+1}$ $H = \frac{wh^2}{4} \frac{k\{h(k+3)+2f\}}{\beta}$ $M_A = \frac{wh^2}{24} \left\{ \frac{12k+6}{3k+1} + \frac{kh^2(k+6)+kf(15h+16f)+6f^2}{\beta} \right\}$ $M_E = \frac{wh^2}{24} \left\{ \frac{12k+6}{3k+1} - \frac{kh^2(k+6)+kf(15h+16f)+6f^2}{\beta} \right\}$

荷重状態	反力
	$V = \frac{P}{2}$ $H = \frac{P\ell}{4} \frac{k(3h+4f)+f}{\beta}$ $M = \frac{P\ell}{4} \frac{kh^2+hf(2k+1)}{\beta}$
	$V_A = P(1-\varepsilon) \frac{3k+(1-\varepsilon)(1+2\varepsilon)}{3k+1}$ $V_E = P\varepsilon \frac{3k+\varepsilon(3-2\varepsilon)}{3k+1}$ $H = P \frac{\varepsilon\ell}{\beta} \{3k(h+f) - 4\varepsilon^2(k+1)f - 3\varepsilon(kh-f)\}$ $M_A = \frac{P\varepsilon\ell}{2} \left[\frac{1}{\beta} \{2k(1-\varepsilon)h^2 + 3(2\varepsilon+k)hf - (1-4\varepsilon)f^2 - 4\varepsilon^2(k+2)hf - 4\varepsilon^2 f^2\} - \frac{(1-\varepsilon)(1-2\varepsilon)}{3k+1} \right]$ $M_E = \frac{P\varepsilon\ell}{2} \left[\frac{1}{\beta} \{2k(1-\varepsilon)h^2 + 3(2\varepsilon+k)hf - (1-4\varepsilon)f^2 - 4\varepsilon^2(k+2)hf - 4\varepsilon^2 f^2\} + \frac{(1-\varepsilon)(1-2\varepsilon)}{3k+1} \right]$
	$V = \frac{Ph}{\ell} \frac{3k}{3k+1}, \quad H = P$ $M_A(M_E) = \frac{Ph}{2} \frac{3k+2}{3k+1}, \quad M_B(M_D) = \frac{Ph}{2} \frac{3k}{3k+1}$ $\delta = \frac{Ph^2}{12EK_1} \frac{3k+4}{3k+1}$
	$V_E = \frac{3Pa^2}{2h\ell} \frac{k}{3k+1}, \quad H = \frac{Pa^2 k}{2h} \frac{3h(k+2)+3f-2a(k+1)}{\beta}$ $M_A = \frac{Pa}{2h} \left\{ -\frac{h^2 k(4h+hk-2ak-6a+6f)+a^2 k(hk+2h+f)}{\beta} + \frac{2fk(2hf-af-3ah)+hf^2}{\beta} - \frac{2h+3k(2h-a)}{6k+2} \right\}$ $M_E = \frac{Pa}{2h} \left\{ -\frac{h^2 k(4h+hk-2ak-6a+6f)+a^2 k(hk+2h+f)}{\beta} + \frac{2fk(2hf-af-3ah)+hf^2}{\beta} + \frac{2h+3k(2h-a)}{6k+2} \right\}$
	$V_E = \frac{3Pxk}{h\ell} \frac{a}{3k+1}, \quad V_A = P - V_E, \quad H = \frac{3Pxk}{h} \frac{a\{h+f+b(k+1)\}}{\beta}$ $M_A = \frac{Px}{2h} \left\{ \frac{h^2 k(2bk+2h+3f)-bfk(6h+3b+4f)}{\beta} - \frac{h(3b^2 k^2 + 6b^2 k + f^2)}{\beta} - \frac{3bk+h}{3k+1} \right\}$ $M_E = \frac{Px}{2h} \left\{ \frac{h^2 k(2bk+2h+3f)-bfk(6h+3b+4f)}{\beta} - \frac{h(3b^2 k^2 + 6b^2 k + f^2)}{\beta} + \frac{3bk+h}{3k+1} \right\}$

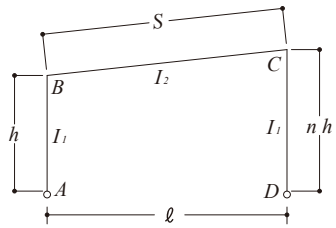
柱脚ピン門形ラーメン

	$K_1 = \frac{I_1}{h}$ $K_2 = \frac{I_2}{l}$ $k = \frac{K_2}{K_1}$
荷重状態	反力
	$V = \frac{w\ell}{2}$ $H = \frac{w\ell^2}{4h} \frac{1}{2k+3}$
	$V_A = (1-\varepsilon)P$ $V_D = \varepsilon P$ $H = \frac{3P\ell}{2h} \frac{\varepsilon(1-\varepsilon)}{2k+3}$ <p>$\varepsilon = \frac{1}{2}$ のとき $V_A = V_D = \frac{P}{2}$</p> $H = \frac{3P\ell}{8h} \frac{1}{2k+3}$
	$V = \frac{wh^2}{2\ell}$ $H = wh \frac{5k+6}{8(2k+3)}$
	$V = \frac{Ph}{\ell}$ $H = \frac{P}{2}$

柱脚固定門形ラーメン

	$K_1 = \frac{I_1}{h}$ $K_2 = \frac{I_2}{l}$ $k = \frac{K_2}{K_1}$
荷重状態	反力
	$V = \frac{w\ell}{2}$ $H = \frac{w\ell^2}{4h} \frac{1}{k+2}$ $M = \frac{w\ell^2}{12} \frac{1}{k+2}$
	$V_A = P(1-\varepsilon) \frac{6k+1+\varepsilon(1-2\varepsilon)}{6k+1}$ $V_D = P - V_A$ $H = \frac{3P\ell}{2h} \frac{\varepsilon(1-\varepsilon)}{k+2}$ $M_A = \frac{P\ell}{2} \varepsilon(1-\varepsilon) \frac{5k-1+2\varepsilon(k+2)}{(k+2)(6k+1)}$ $M_D = \frac{P\ell}{2} \varepsilon(1-\varepsilon) \frac{7k+3-2\varepsilon(k+2)}{(k+2)(6k+1)}$
	$V = \frac{wh^2}{\ell} \frac{k}{6k+1}$ $H = \frac{wh}{8} \frac{2k+3}{k+2}$ $M_A = \frac{wh^2}{24} \left(12 - \frac{5k+9}{k+2} - \frac{12k}{6k+1} \right)$ $M_D = \frac{wh^2}{24} \left(\frac{5k+9}{k+2} - \frac{12k}{6k+1} \right)$
	$V = \frac{Ph}{\ell} \frac{3k}{6k+1}$ $H = \frac{P}{2}$ $M_A = \frac{Ph}{2} \frac{3k+1}{6k+1}$

柱脚ピン片流れ門形ラーメン



$$K_1 = \frac{I_1}{h}$$

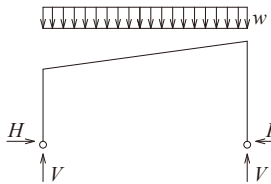
$$K_2 = \frac{I_2}{S}$$

$$k = \frac{K_2}{K_1}$$

$$\gamma = 1 + n + n^2 + (1 + n^3)k$$

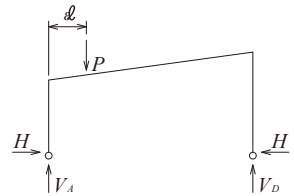
荷重状態

反力



$$V = \frac{w\ell}{2}$$

$$H = \frac{w\ell^2}{8h} \frac{1+n}{\gamma}$$



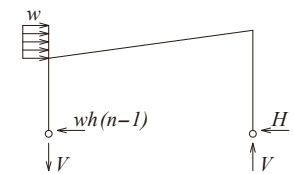
$$V_A = (1 - \varepsilon)P$$

$$V_D = \varepsilon P$$

$$H = \frac{P\ell\varepsilon(1 - \varepsilon)}{2h} \frac{2 + n + \varepsilon(n - 1)}{\gamma}$$

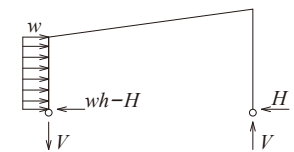
$$\varepsilon = \frac{l}{2} \text{ のとき } V_A = V_D = \frac{P}{2}$$

$$H = \frac{3P\ell}{16h} \frac{n+1}{\gamma}$$



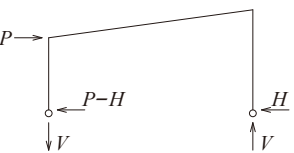
$$V = \frac{wh^2}{2\ell} (n^2 - 1)$$

$$H = \frac{wh(n-1)}{8} \frac{8k + 7 + n(n+4)}{\gamma}$$



$$V = \frac{wh^2}{2\ell}$$

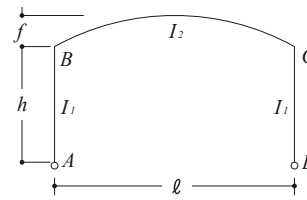
$$H = \frac{wh}{8} \frac{5k + 2(n+2)}{\gamma}$$



$$V = \frac{Ph}{\ell}$$

$$H = \frac{P}{2} \frac{2k + n + 2}{\gamma}$$

柱脚ピンアーチ形ラーメン



$$K_1 = \frac{I_1}{h}$$

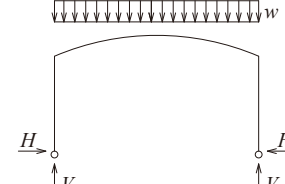
$$K_2 = \frac{I_2}{\ell}$$

$$k = \frac{K_2}{K_1}$$

$$\delta = 5h^2(2k+3) + 4f(5h+2f)$$

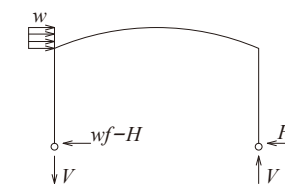
荷重状態

反力



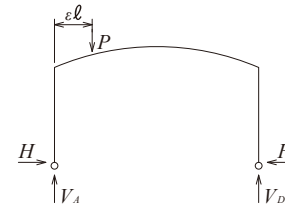
$$V = \frac{w\ell}{2}$$

$$H = \frac{w\ell^2}{4} \frac{5h+4f}{\delta}$$



$$V = \frac{wf}{2\ell} (2h+f)$$

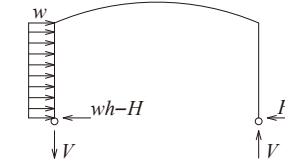
$$H = \frac{wf}{14} \frac{35h^2(2k+3) + 16f(7h+2f)}{\delta}$$



$$V_A = (1 - \varepsilon)P$$

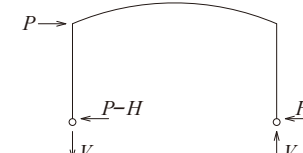
$$V_D = \varepsilon P$$

$$H = \frac{5P\ell}{2} \frac{3h + 2f\{\varepsilon(1 - \varepsilon) + 1\}}{\delta} \varepsilon(1 - \varepsilon)$$



$$V = \frac{wh^2}{2\ell}$$

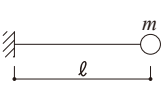
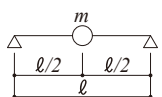
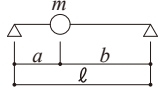
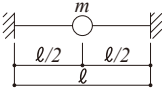
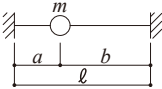
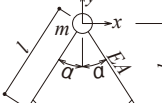
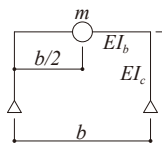
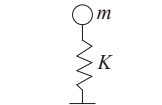
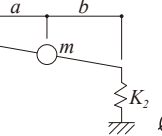
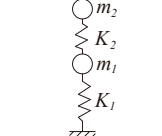
$$H = \frac{5wh^2}{8} \frac{h(5k+6) + 4f}{\delta}$$



$$V = \frac{Ph}{\ell}$$

$$H = \frac{5Ph}{2} \frac{h(2k+3) + 2f}{\delta}$$

3-3. 各種構造物固有円振動数

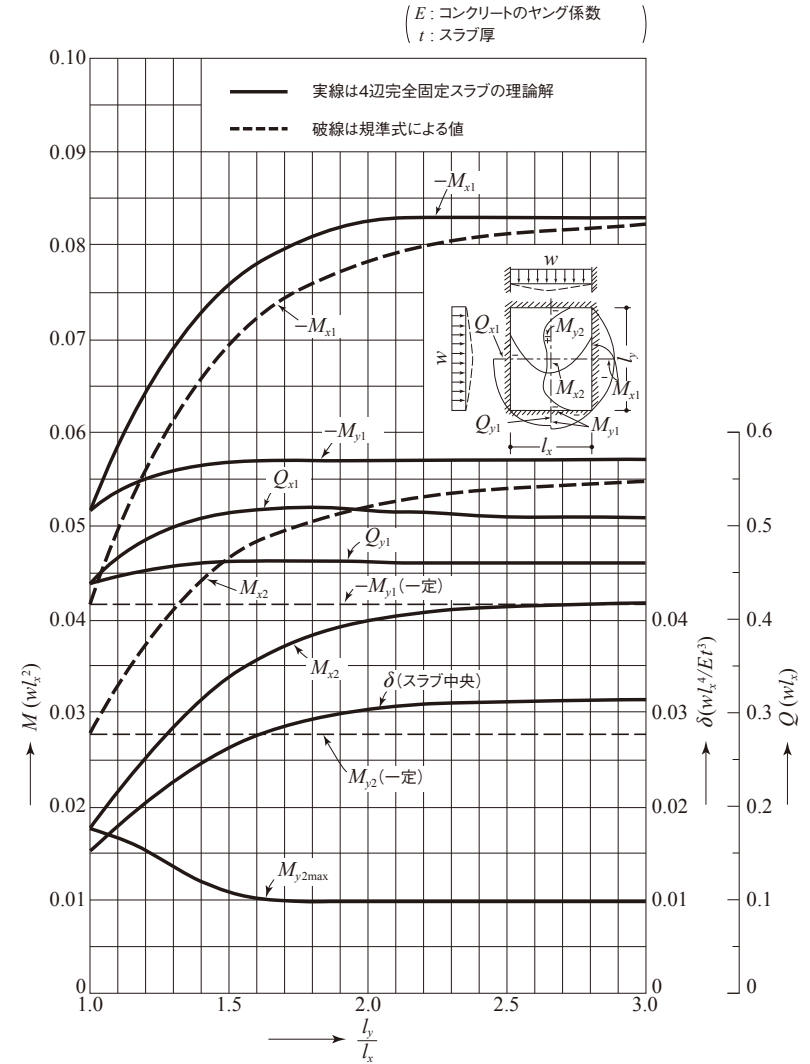
	$K=3EI/l^3$ $\omega_n = \sqrt{\frac{3EI}{m l^3}}$ はりの質量 m_b を考えるとき $\omega_n' = \sqrt{\frac{3EI}{(m+0.23m_b)l^3}}$
	$K=48EI/l^3$ $\omega_n = 4 \times \sqrt{\frac{3EI}{m l^3}}$ はりの質量 m_b を考えるとき $\omega_n' = \sqrt{\frac{48EI}{(m+0.5m_b)l^3}}$
	$K = \frac{3EI\ell}{(ab)^2}$ $\omega_n = \frac{1}{ab} \times \sqrt{\frac{3EI\ell}{m}}$
	$K = \frac{192EI}{l^3}$ $\omega_n = 8 \times \sqrt{\frac{3EI}{m l^3}}$ はりの質量 m_b を考えるとき $\omega_n' = 14 \times \sqrt{\frac{EI}{(m+0.375m_b)l^3}}$
	$K = \frac{3EI\ell^3}{(ab)^3}$ $\omega_n = \sqrt{\frac{3EI\ell^3}{a^3 b^3 m}}$
	$K_x = \frac{2EA}{l} \sin^3 \alpha$ $\omega_n = \sqrt{\frac{K_x}{m}}$ $K_y = \frac{2EA}{l} \cos^3 \alpha$ $\omega_n = \sqrt{\frac{K_y}{m}}$
	$K_x = \frac{bEI_c}{h^3} \times \frac{1}{1 + \frac{I_c b}{2I_b h}}$ $\omega_n = \sqrt{\frac{K_x}{m}}$ $K_y = \frac{192EI_b}{h^3} \times \frac{1 + \frac{2I_b b}{3I_c h}}{1 + \frac{8I_b h}{3I_c b}}$ $\omega_n = \sqrt{\frac{K_y}{m}}$
	$\omega_n = \sqrt{\frac{K}{m}}$ はねの質量 m' を考えるとき $\omega_n = \sqrt{\frac{K}{(m+m'/3)}}$
	$\omega_n = \sqrt{\frac{K_n}{m}}$ $\frac{1}{K_n} = \frac{1}{K_1} \left(\frac{b}{\ell}\right)^2 + \frac{1}{K_2} \left(\frac{a}{\ell}\right)^2$
	$\omega_n^2 = \frac{1}{2} \left[\frac{K_1}{m_1} + \frac{K_2}{m_2} \left(1 + \frac{m_2}{m_1}\right) \pm \sqrt{\left\{ \frac{K_1}{m_1} + \frac{K_2}{m_2} \left(1 + \frac{m_2}{m_1}\right) \right\}^2 - \frac{4K_1 K_2}{m_1 m_2}} \right]$

K:ばね定数 m:質量 ω_n :固有振動数 EI:曲げ剛性

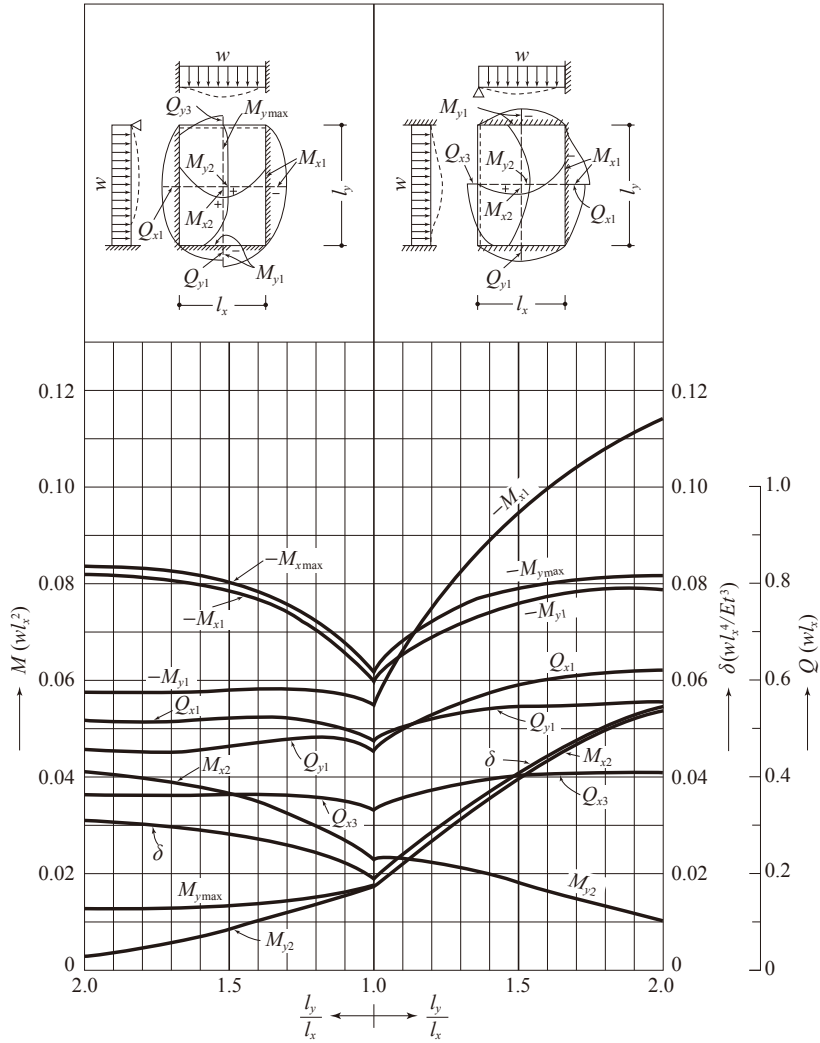
3-4. 長方形スラブ算定図

長方形スラブの応力とたわみ

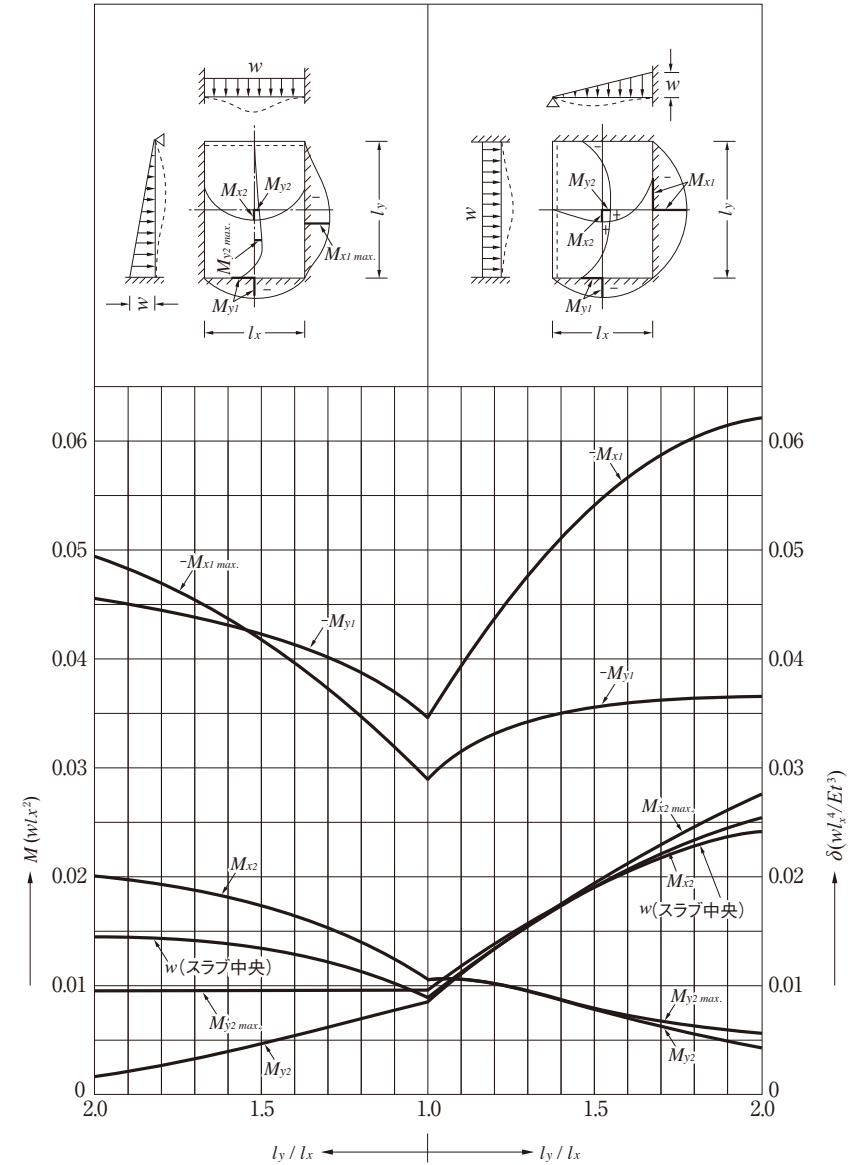
等分布荷重時4辺固定スラブの応力図と中央点のたわみ($v=0$)



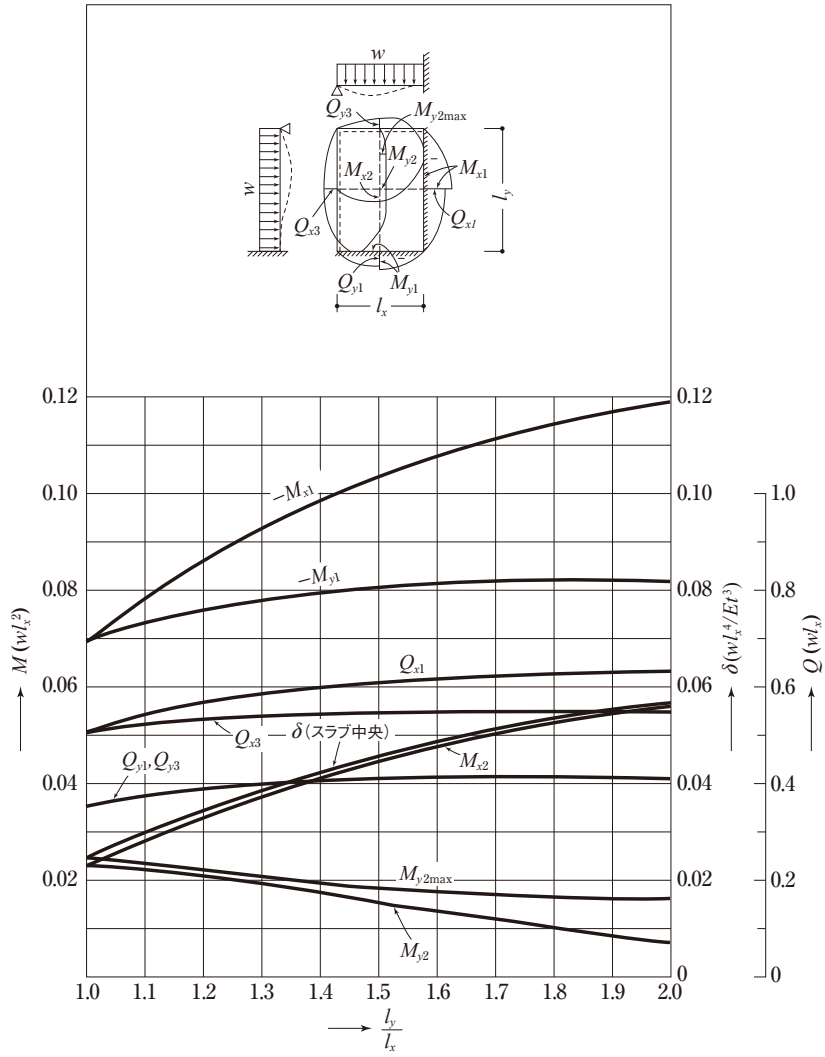
等分布荷重時3辺固定1辺単純支持スラブの応力図と中央点のたわみ($\nu=0$)



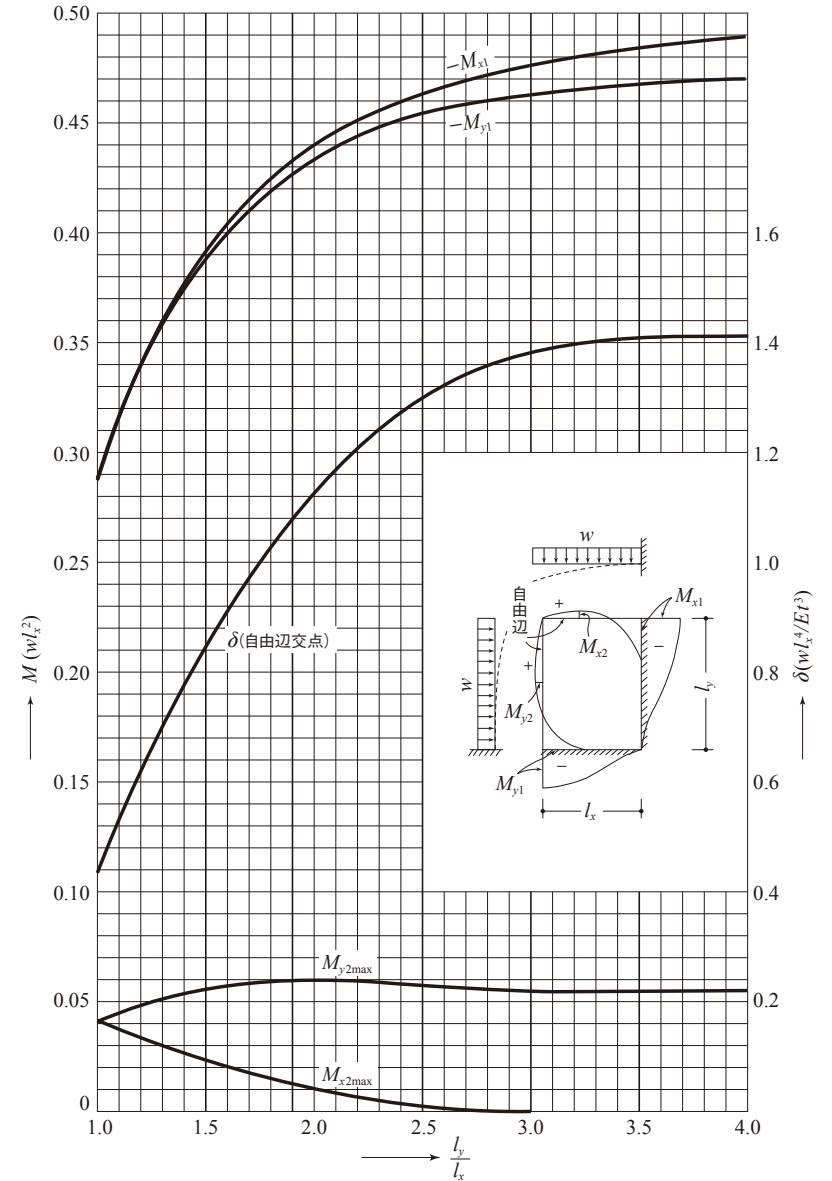
等変分布荷重時3辺固定、1辺単純支持スラブの応力図と中央点のたわみ($\nu=0$)



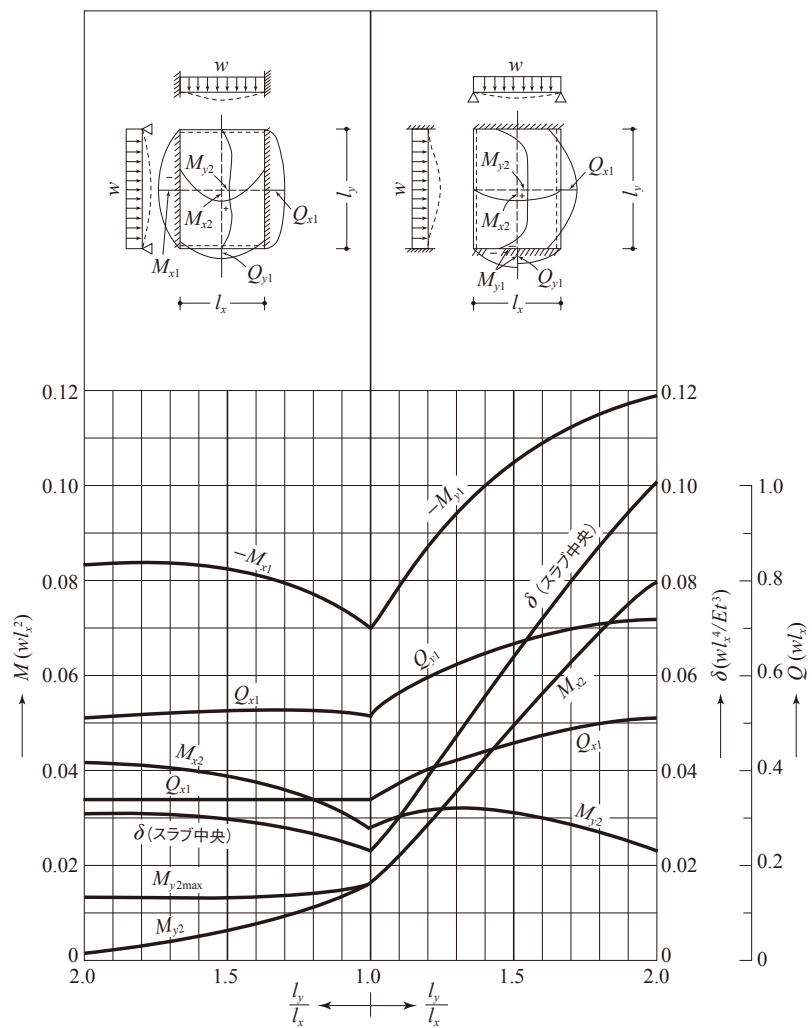
等分布荷重時2隣辺固定2辺単純支持スラブの応力図と中央点のたわみ($\nu=0$)



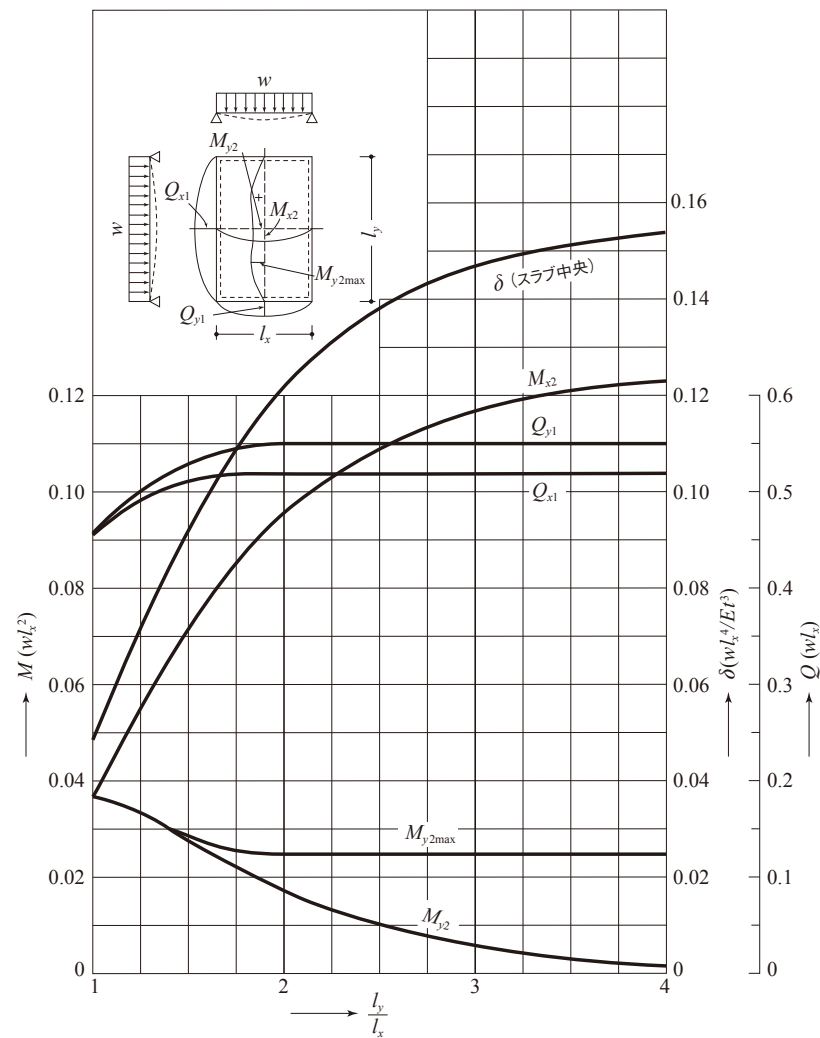
等分布荷重時2隣辺固定他辺自由スラブの応力図と自由辺交点のたわみ($\nu=0$)



等分布荷重時2対辺固定他辺単純支持スラブの応力図と中央点のたわみ ($\nu=0$)



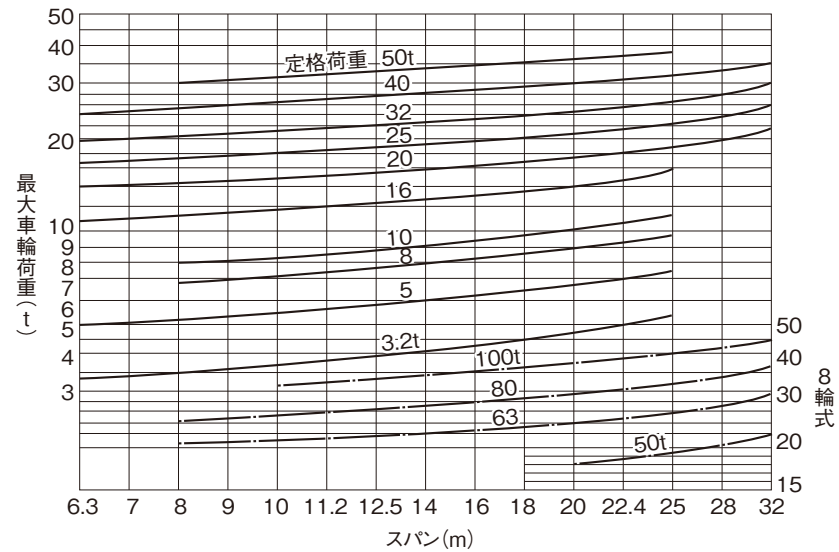
等分布荷重時4辺単純支持スラブの応力図と中央点のたわみ ($\nu=0$)



クレーン自重(建築学会・建築物荷重指針より)

(単位:t)

定格荷重(t)		スパン(m)								
主巻	補巻	8	10	12	14	16	18	20	22	24
3	-	7.2	8.1	9.0	9.9	10.9	11.8	12.8	13.8	14.8
5	-	8.8	9.7	10.6	11.5	12.6	13.7	14.7	15.8	16.9
10	-	11.6	12.8	14.0	15.3	16.5	17.7	19.2	21.0	22.8
15	3		16.9	18.4	19.9	21.4	22.9	24.4	26.0	27.5
25	5			25.3	27.3	29.3	31.4	33.5	35.5	37.6
30	5			27.1	29.4	31.2	34.0	36.3	38.6	40.8
40	10			38.4	41.1	43.8	46.5	48.9	51.3	53.7
50	10			46.8	50.3	53.7	57.1	60.5	64.3	68.1
60	10			54.2	58.9	63.5	68.0	71.7	75.4	79.1
80	20			70.3	74.5	78.7	83.0	88.1	93.1	98.2
100	20			86.0	91.0	96.0	101.0	107.7	114.5	121.2



最大車輪荷重(JIS B8801-1974)

備考)一点鎖線は8輪式を示す。

